An Overview of Algorithms for Downlink Transmit Beamforming

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Abstract Downlink beamforming refers to the problem of using an array of antennas at a particular node (e.g., a basestation) in a wireless network to communicate simultaneously with multiple co-channel users. The users in the network may have a single antenna, and hence no ability for spatial discrimination, or they may have multiple antennas and the ability to perform some type of interference suppression. The primary issue is how to balance the need for high, received signal power for each user against the interference produced by the signal at other points in the network. In this presentation, we describe several approaches to this problem: channel inversion, regularized channel inversion, channel block diagonalization, coordinated transmit/receive beamforming, and dirty-paper coding. While the basic idea behind these algorithms is the same, namely the use of channel information at the transmitter to predict and then counteract the interference produced at each node in the network, each of the algorithms is based on achieving a different performance objective. Typical performance criteria include zero-interference transmission, minimum transmit power subject to a minimum signal-to-interference plus noise ratio at each receiver, or maximum throughput subject to a given transmit power constraint. We compare the various goals of the above algorithms, and detail their respective advantages and disadvantages in terms of computational complexity, required transmit power, network throughput, and assumed receiver capabilities. The results of several simulation studies are presented to quantify these comparisons.

Report Documentation Page	Form Approved OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
20 DEC 2004	**N/A**	**-**

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
An Overview of Algorithms for Downlink Transmit Beamforming	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Brigham Young University**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also, ADM001741 Proceedings of the Twelfth Annual Adaptive Sensor Array Processing Workshop, 16-18 March 2004 (ASAP-12, Volume 1)., The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**29**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

An Overview of Algorithms for Downlink Transmit Beamforming

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Presentation Outline

- Background
  - Single-User vs. Multi-User MIMO Scenarios
  - Mathematical Notation
- Algorithms for Single-Antenna Users
  - Channel Inversion
  - Regularized Channel Inversion
  - Vector Modulo Pre-Coding
  - Interference-Balancing Methods for Power Control
- Algorithms for Multiple-Antenna Users
  - Joint Transmit/Receive Beamforming
  - One vs. Multiple Sub-Channels per User
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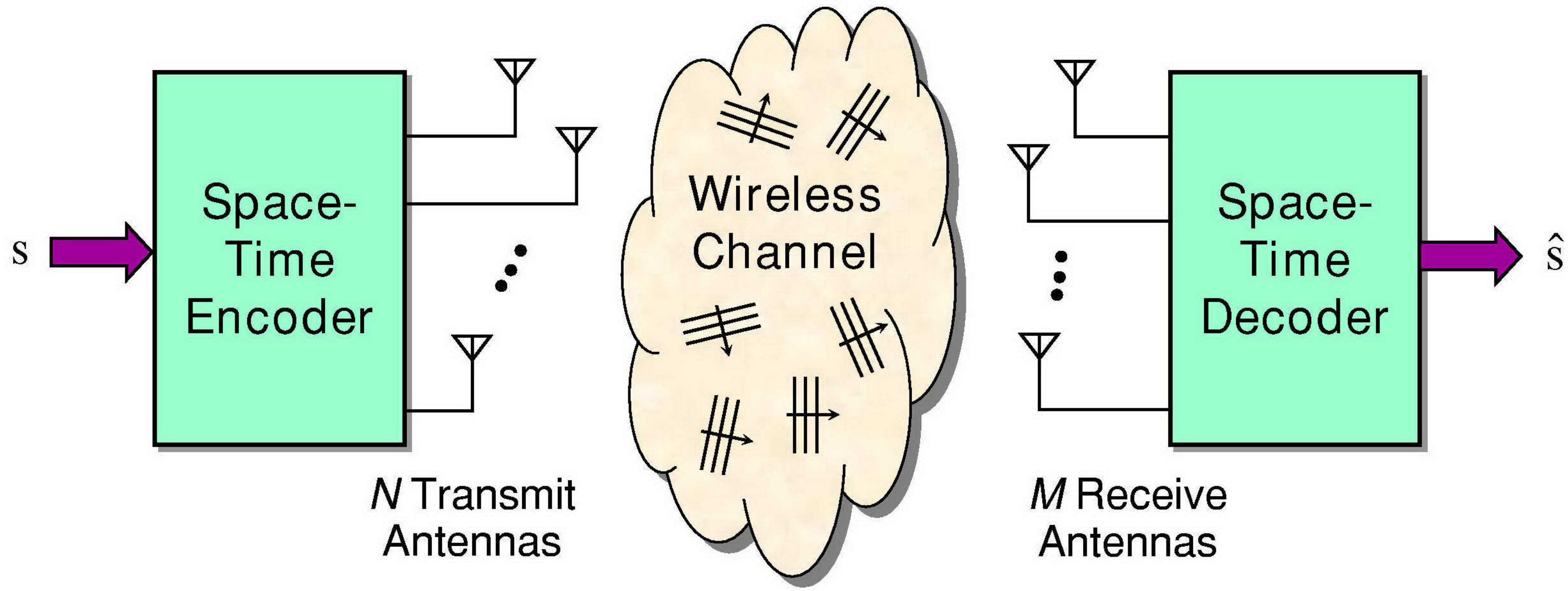
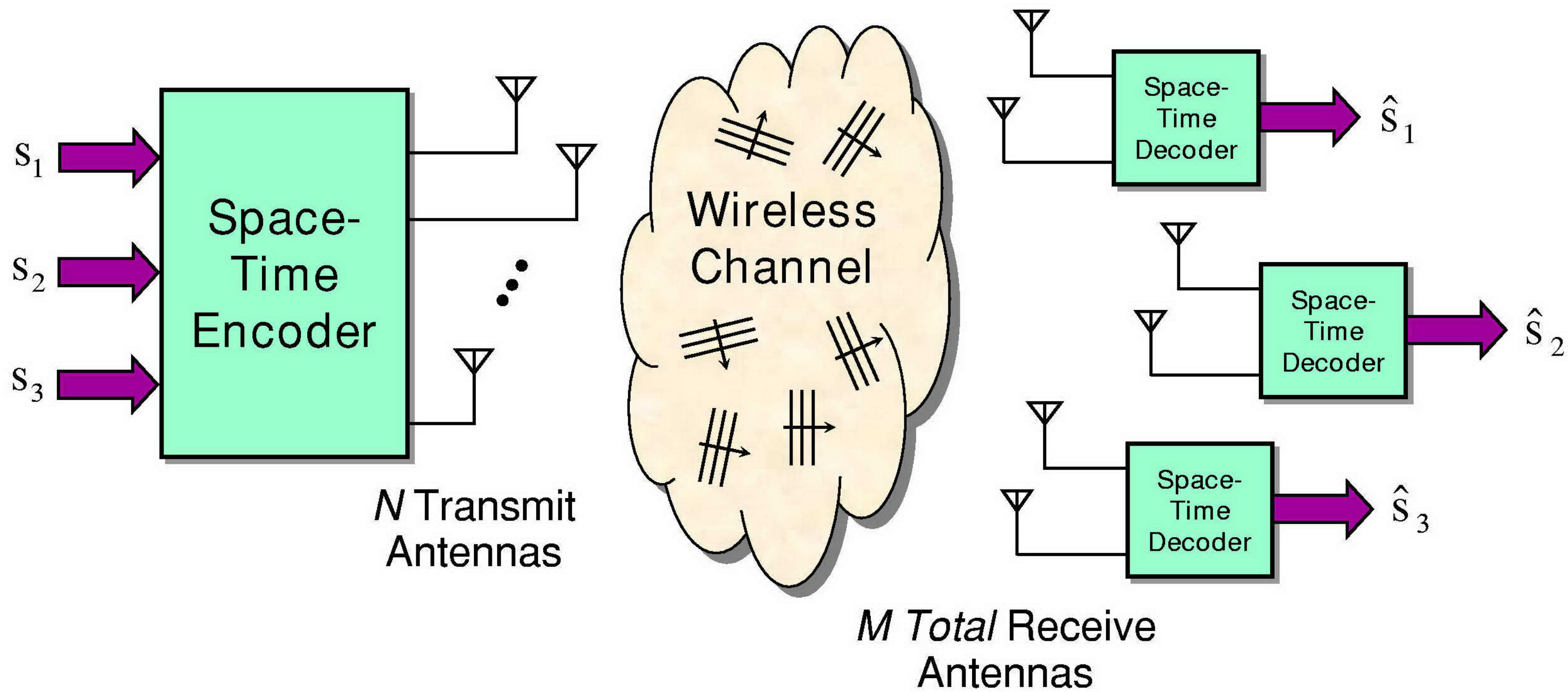
Single-User, Point-to-Point MIMO



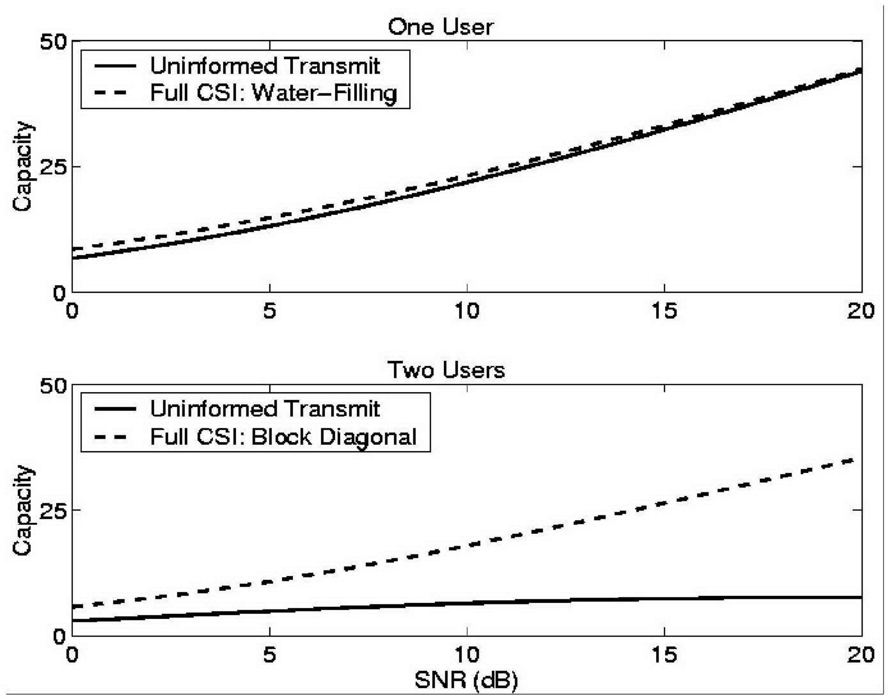
- Receive processing is centralized, assume CSI at receiver (rCSI)
- Under ideal conditions (e.g., independent Rayleigh fading)
  - capacity grows linearly with min(N,M)
  - capacity growth is independent of CSI at the transmitter (xCSI)
- If channel is rank deficient, xCSI is much more important

Multi-User MIMO Downlink



- Receive processing is distributed, only local rCSI is available
- Without xCSI, no capacity growth at high SNR due to interference, even in the ideal case

MU-MIMO Capacity Example



Mathematical Notation

Assume P users, each with m antennas, d data streams transmitted to each user $(m \geq d)$.
Signal received by user 1:

$$x_1 = \underbrace{H_1}_{m \times N \text{ sub-channel matrix}} \underbrace{T_1}_{N \times d \text{ transmit beamformer}} s_1 + \sum_{k=2}^{P} \underbrace{H_k T_k s_k}_{\text{data intended for other users}} + \underbrace{n_1}_{\text{Noise and unmodeled interference}}$$

Assume each user's data stream is scaled so that $\|s_i\| = 1$.

System equation:

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_P \end{bmatrix} = \begin{bmatrix} H_1 \\ \vdots \\ H_P \end{bmatrix} \begin{bmatrix} T_1 & \cdots & T_P \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_P \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_P \end{bmatrix}$$

$$= HTs + n$$

Total # of receive antennas: $M = mP$

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Special Case: Single Antenna Users

Assume each user has $m = 1$ antenna, and $N \geq M = P$.

Channel Inversion

Transmitter "pre-inverts" the channel. Transmit beamformers are columns of the pseudo-inverse:

$$\boxed{T_{ci} = \gamma H^* \left(HH^*\right)^{-1}}$$

To maintain fixed transmit power ρ, must scale signal:

$$\gamma = \sqrt{\frac{\rho}{s^* \left(HH^*\right)^{-1} s}}$$

Ideally, channel inversion eliminates all inter-user interference:

$$x = HT_{ci} s + n = \gamma HH^* \left(HH^*\right)^{-1} s + n = \gamma s + n$$

- problems obviously arise when channel is (nearly) rank deficient
- problem isn't *noise amplification,* instead it is *signal attenuation* due to power constraints
- what about in the "ideal" case, e.g., with independent Rayleigh fading?

Capacity of Channel Inversion

Assume elements of H are independent, Rayleigh w/ unit variance, $N = P = M$ and $s \sim \text{Q} \; (0, I)$.

A bad sign:

$$\lambda = s^*\left(HH^*\right)^{-1} s \quad \text{is distributed as*} \quad p(\lambda) = N \frac{\lambda^{N-1}}{(1+\lambda)^{N+1}}$$

$$\text{and} \quad E(\lambda) = \infty \quad !!$$

*Capacity** for large N:*

$$\boxed{\lim_{N \to \infty} C_{ci} = \frac{\rho}{\sigma^2} \log_2(e)}$$

$\Longrightarrow$ No capacity growth with # of users/antennas

* Hochwald & Vishwanath, *Proc. 40th Allerton Conf.*, October, 2002

** Peel, Hochwald & Swindlehurst, *Proc. 41st Allerton Conf.*, October 2003

Regularized Channel Inversion

A simple fix is to regularize the inverse:

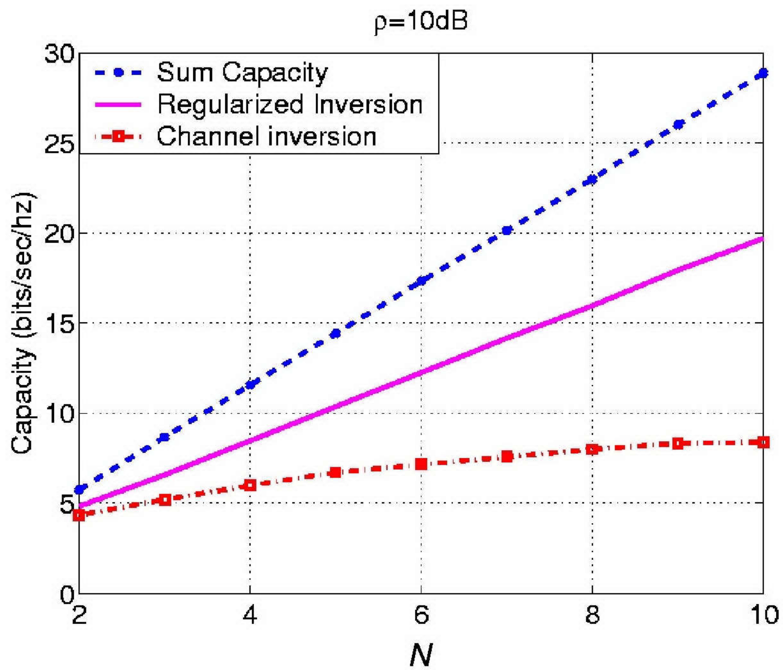
$$T_{rci} = \gamma H^* \left(HH^* + \alpha I \right)^{-1}$$

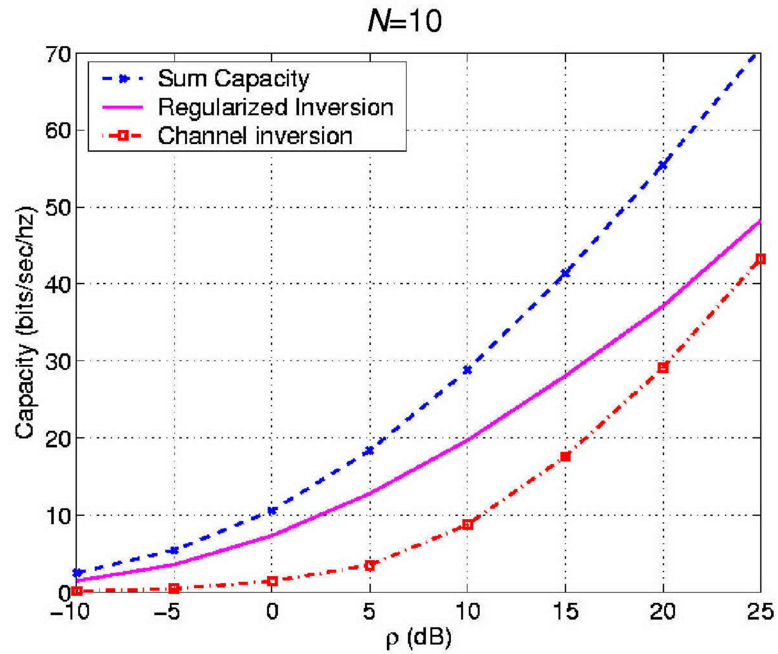
(To maximize SINR @ the receivers, choose*

$$\alpha = \frac{N\sigma^2}{\rho}$$

*Peel, Hochwald, Swindlehurst: “A vector perturbation technique for near-capacity multi-antenna, multi-user communication, Part I”, submitted to *IEEE Trans. Comm.)*

Linear growth with N is recovered:





What's the real issue?

- The regularization "quick fix" helps, but there is still a significant performance gap
- Ultimately, problems arise when s happens to lie in the direction of the "largest" singular vector of $(HH^*)^{-1}$
- *IDEA:* could we perturb s to eliminate this possibility, *i.e.*,

$$\boxed{\min_{s_p} \left(s+s_p\right)^*\left(HH^*\right)^{-1}\left(s+s_p\right)}$$

and still decode s at the receivers, without knowledge of s_p ?

- Yes, using a little "trick": *modulo pre-coding**

*M. Tomlinson, "New automatic equaliser employing modulo arithmetic," *Elec. Letters*, March, 1971
H. Harashima & H. Miyakawa, "Matched transmission technique for channels with intersymbol interference," *IEEE Trans. Comm.,* August, 1972

Vector Modulo Pre-coding

Use a perturbation of the form

$$s_p = \tau c = \tau(a + jb)$$

where τ is real and *a,b* are vectors of integers. For channel inversion:

$$x = HT_{ci}(s + s_p) + n = \gamma HH^*(HH^*)^{-1}(s + s_p) + n$$
$$= \gamma s + \gamma \tau c + n$$

Assuming receivers know γ, perfect decoding is possible w/out noise using the *mod*-function:

$$f_\tau(y) = y - \left\lfloor \frac{y + \tau/2}{\tau} \right\rfloor \tau$$

(*mod*-function applied to real & imaginary parts separately)

At receiver *k*,

$$f_\tau\left(\frac{1}{\gamma} x_k\right) = f_\tau(s_k + \tau a_k + \tau j b_k) = s_k$$

τ must be chosen large enough to avoid *mod*-function ambiguities: $\tau = 2(d_{\max} + \Delta/2)$,

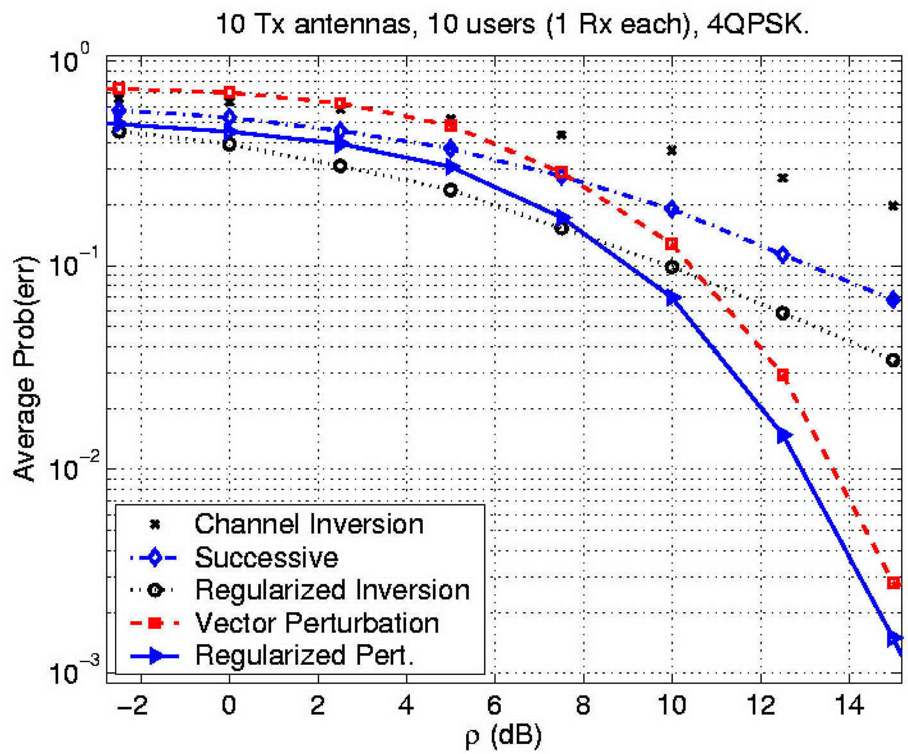
$d_{\max}$ = constellation "size"
Δ = constellation "spacing"

Vector Modulo Pre-coding (cont.)

- Choose c to solve integer-lattice least-squares problem (sphere *encoding*):

$$\min_{a \in L, b \in L} \left(s+\tau a+\tau j b\right)^{*}\left(HH^{*}\right)^{-1}\left(s+\tau a+\tau j b\right)$$

- $\tau \to 0, \infty \;\Rightarrow$ standard ch. inversion
- can regularize this approach too
- provides linear capacity growth w/ N
- with outer turbo code, this approach gets within 3-4 dB of capacity
- to get closer, one must resort to "dirty paper" techniques, which lead to much more complex receivers



An Alternative Based on Power Control (Interference Balancing)

With a single antenna at each receiver, the columns of $T = [\ t_1 \ \ldots\ t_P\ ]$ represent the transmit beamformers for each user, and each channel is a row vector $H_i = h_i^*$:

$$x_i = h_i^* t_i s_i + \sum_{k \neq i} h_k^* t_k s_k + n_i$$

interference from signals sent to other users

In the *power control* formulation, minimize total transmitted power subject to a certain QoS constraint, usually measured by SINR:

$$\min \sum_{k=1}^{P} t_k^* t_k \qquad \text{s.t.} \qquad \frac{t_i^* h_i h_i^* t_i}{\sum_{k \neq i} t_k^* h_k h_k^* t_k + \sigma^2} \geq \beta_i \ , \qquad i = 1, \cdots, P$$

Can be posed as a convex, semi-definite optimization & efficiently solved using (for example) interior-point methods. See

Rashid-Farrokhi, Liu, Tassiulas,
"Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas in Comm.*, October, 1998
Bengtsson & Ottersten
"Optimal and sub-optimal transmit beamforming," *Handbook of Antennas in Wireless Comm.*, CRC Press, August, 2001

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Multiple-Antenna Users

Recall system equation for P users with m antennas each:

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_P \end{bmatrix} = \begin{bmatrix} H_1 \\ \vdots \\ H_P \end{bmatrix} \begin{bmatrix} T_1 & \cdots & T_P \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_P \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_P \end{bmatrix}$$

$$= HTs + n$$

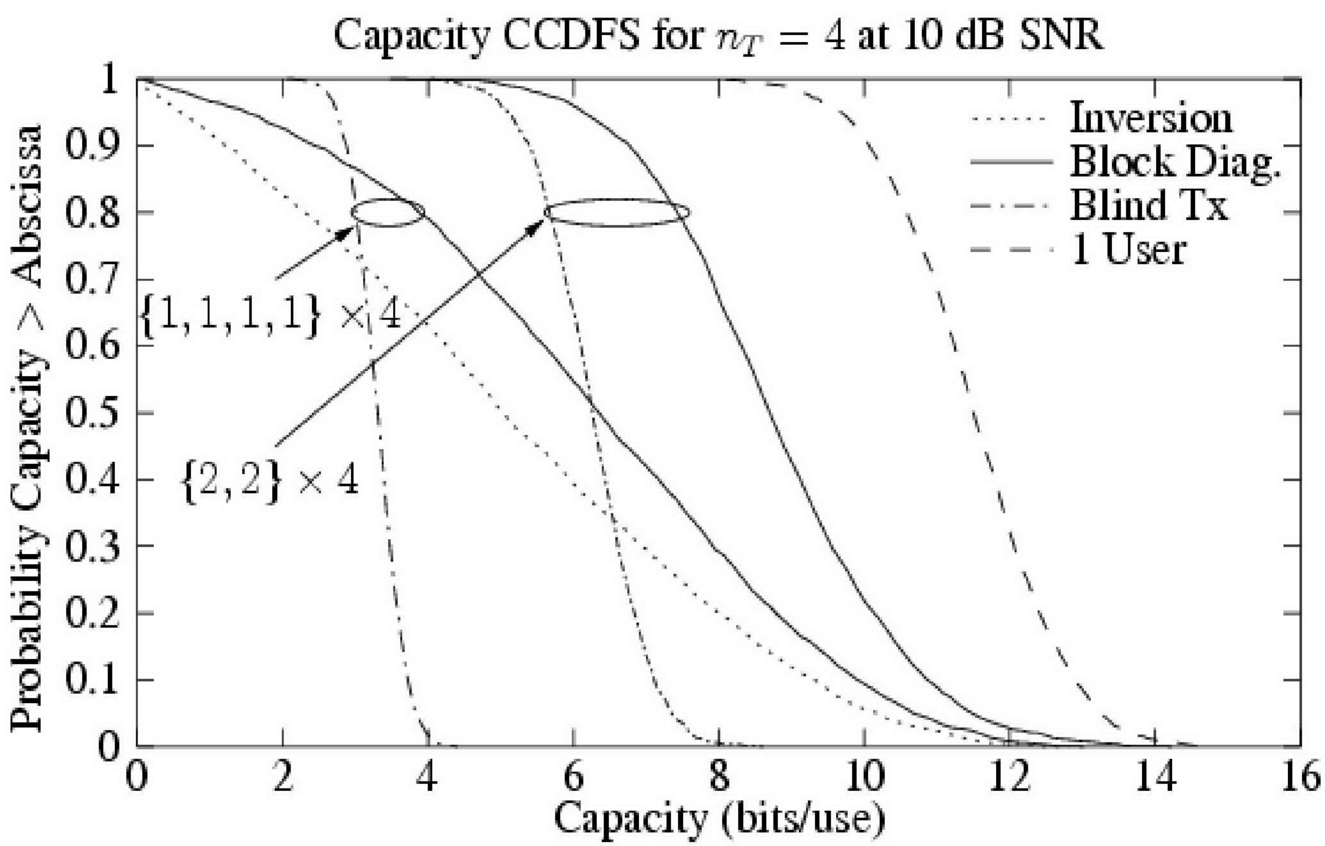
- If $N \geq M = mP$, we could diagonalize *HT*, but such an approach would be sub-optimal, since it would ignore spatial discrimination at the receivers
- From the standpoint of capacity, it is better to *block*-diagonalize *HT*, and then use water-filling to allocate power to all available spatial channels*

*Q. Spencer and A. Swindlehurst, "Zero-forcing methods for downlink spatial multiplexing in multi-user MIMO channels," *IEEE Trans. Sig. Proc.,* February, 2004

- Disadvantage is that capacity may be achieved at the expense of weak users; *e.g.,* 1-2 strong users may take a dominant share of available power

Multiple-Antenna User Example

4 xmit antennas, 4 total rcv antennas, Rayleigh fading channel



Joint Tx-Rx Design Problem: 1 Sub-Channel per User

It is unlikely that $N \geq M = mP$. Can multiplex up to N data streams with N antennas. Assume 1 stream is sent to each of $P \leq N$ users, who employ receive beamformers:

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_P \end{bmatrix} = \begin{bmatrix} w_1^* H_1 \\ \vdots \\ w_P^* H_P \end{bmatrix} \begin{bmatrix} t_1 & \cdots & t_P \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_P \end{bmatrix} + \begin{bmatrix} w_1^* n_1 \\ \vdots \\ w_P^* n_P \end{bmatrix}$$

Composite channel is from xmit antennas to the output of rcv beamformers.

Problem:

- Design of optimal xmit beamformer t_i requires knowledge of all rcv beamformers w_k (see previous slides)
- Design of rcv beamformer w_i requires knowledge of at least the xmit beamformer t_i; for example:

$$w_{\mathrm{MMSE},i} = \left(\sum_{k \neq i} H_k t_k t_k^* H_k^* + \sigma^2 I \right)^{-1} H_i t_i \qquad \text{or} \qquad w_{\mathrm{MRC},i} = H_i t_i$$

Joint Tx-Rx Beamformer Design: 1 Sub-Channel per User

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_P \end{bmatrix} = \begin{bmatrix} w_1^* H_1 \\ \vdots \\ w_P^* H_P \end{bmatrix} \begin{bmatrix} t_1 & \cdots & t_P \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_P \end{bmatrix} + \begin{bmatrix} w_1^* n_1 \\ \vdots \\ w_P^* n_P \end{bmatrix}$$

Consider the following iterative algorithm (at the transmitter):

(1) Find an initial set of rcv weights $w_1 , \ldots , w_P$ (*e.g.,* use singular vectors of the channel matrices)

(2) Calculate xmit beamformers $t_1 , \ldots , t_P$ using desired “single-antenna” algorithm:

- channel inversion
- regularized inversion
- vector precoding
- power control

(3) Calculate optimal receive beamformers using, *e.g.,* MMSE or MRC criteria

(4) Repeat steps 2 and 3 until convergence.

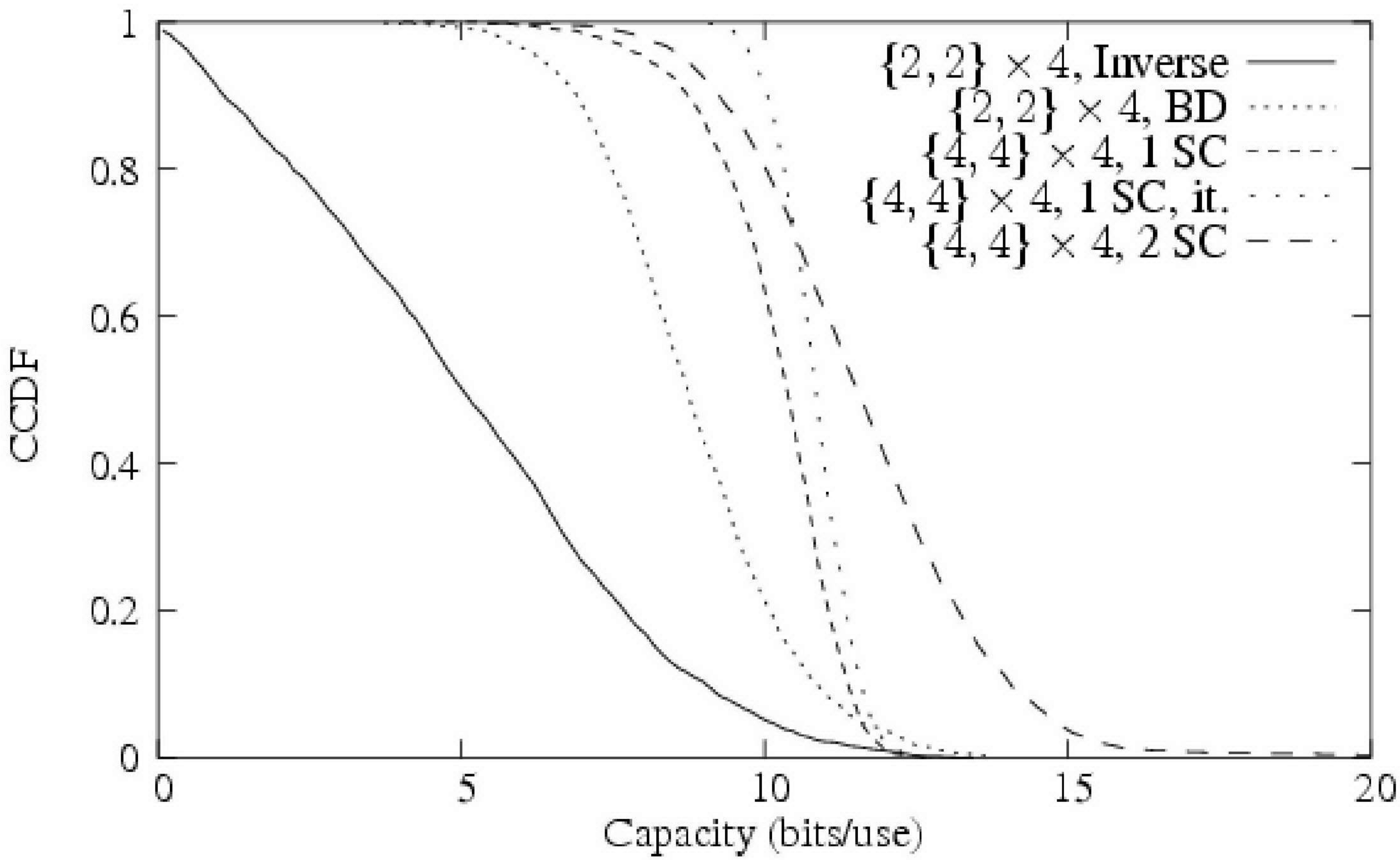
Joint Tx-Rx Beamformer Design: Multiple Sub-Channels per User

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_P \end{bmatrix} = \begin{bmatrix} W_1^* H_1 \\ \vdots \\ W_P^* H_P \end{bmatrix} \begin{bmatrix} T_1 & \cdots & T_P \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_P \end{bmatrix} + \begin{bmatrix} W_1^* n_1 \\ \vdots \\ W_P^* n_P \end{bmatrix}$$

- Total # of sub-channels cannot be greater than # of xmit antennas
- Can employ same algorithms as in single sub-channel case
- Resource allocation is critical: *Who gets the sub-channels?*
  (multi-user diversity)
- Solution should be adaptive; avoid users with rank-deficient channels
  (simulations show fixed allocation strategies perform poorly)
- Larger question: How to group users that are spatially multiplexed?

Joint Tx-Rx Beamformer Example

Rayleigh fading channel



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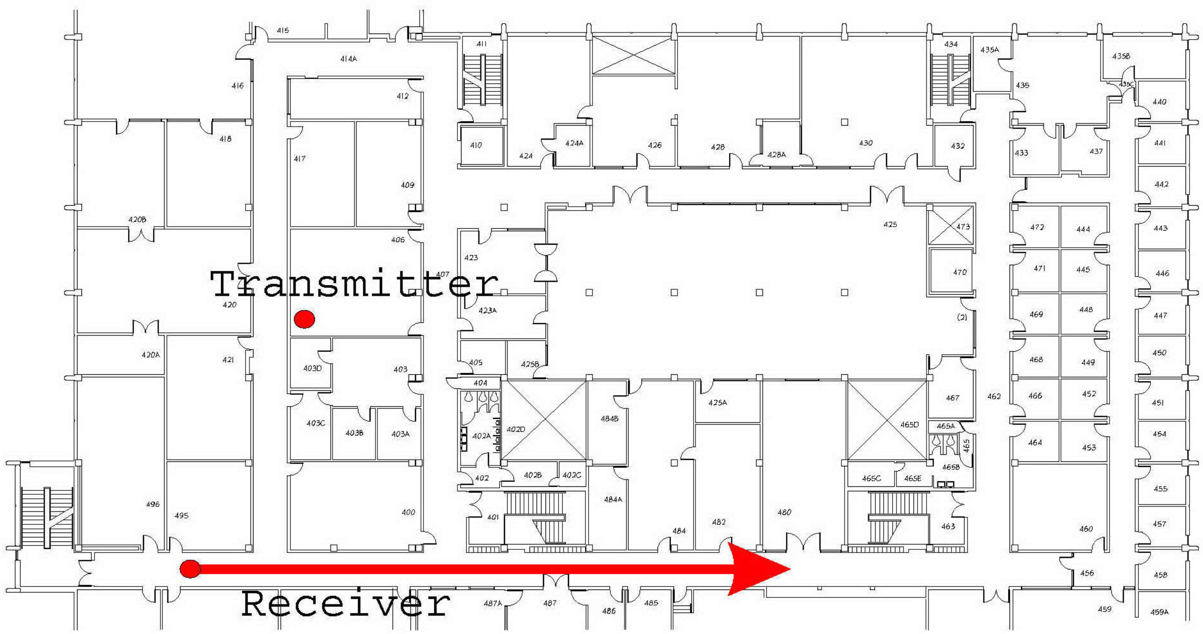
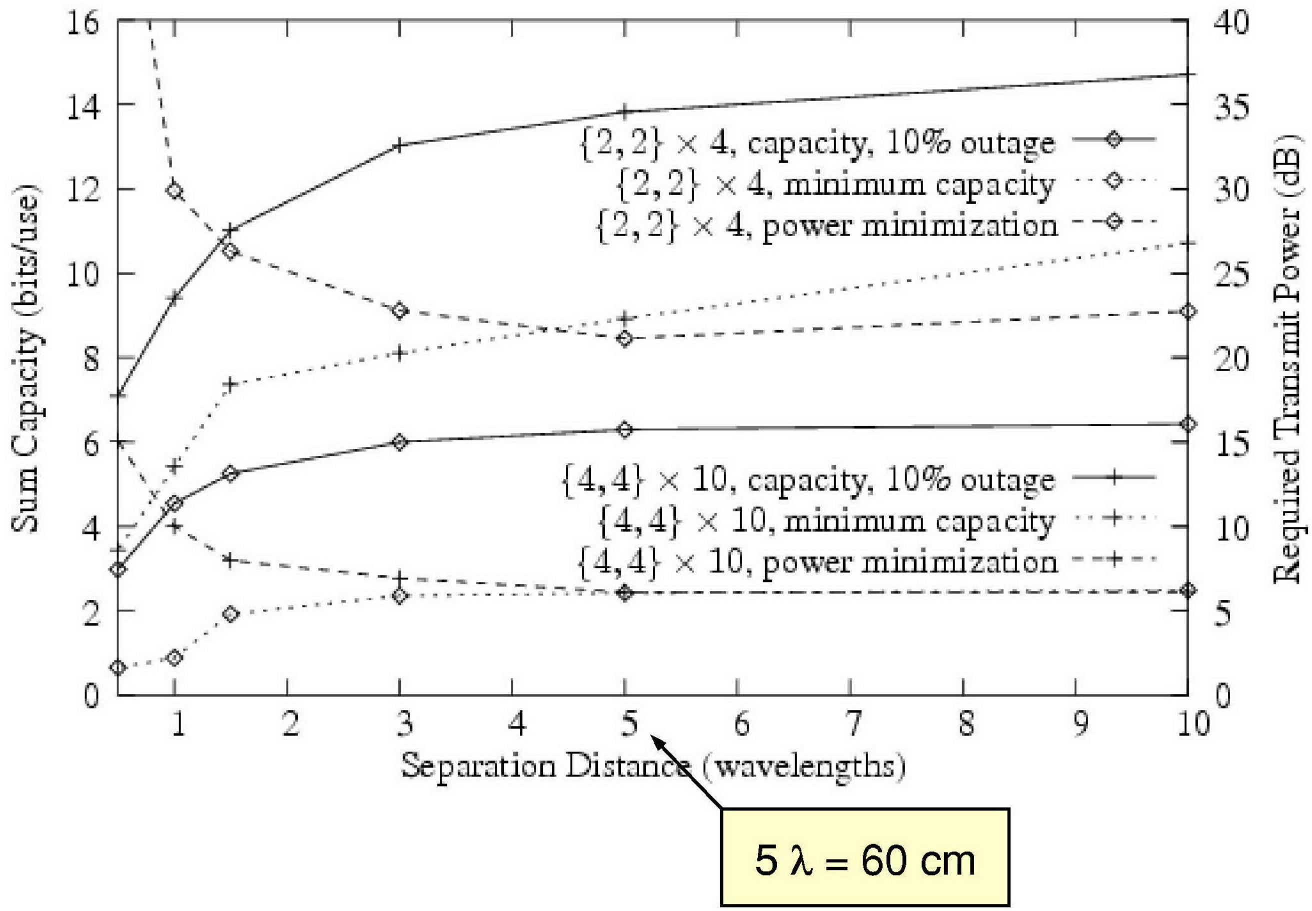
Experimental Results

- 10x10 MIMO channel measurements collected in BYU's Clyde Building
- 2.43 Ghz carrier, 25 kHz bandwidth
- circular transmit and receive arrays, 0.86 λ radius
- transmit array is fixed, receive array is moving
- 29 10x10 channel samples obtained for every λ over 43m.



Experimental Results (cont.)

How close can 2 users be for spatial multiplexing to be possible?



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Summary

- In the multi-user MIMO downlink, must balance desire for high throughput to one user with interference experienced by other users
- Unlike the single-user case, CSI at the transmitter is crucial
- Focus on “closed-form” solutions, simple receiver structures (in general, achieving capacity requires complicated “dirty-paper” techniques and coding schemes)
- Two standard paradigms considered:
  - maximize throughput with zero interference for fixed xmit power
  - minimize xmit power s.t. desired QoS (*e.g.,* rcv SINR) is achieved
- Presented several techniques: *channel inversion, regularized channel inversion, vector modulo pre-coding, interference balancing,* etc.
- Experimental results are promising

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